Name of the Course : **CBCS B.Sc.(H)Mathematics**

Unique Paper Code : **32351101_OC**

Name of the Paper : **C1-Calculus**

Semester : **I**

Duration : **3 Hours**

Maximum Marks : **75**

Attempt any four questions. All questions carry equal marks.

1. Sketch the graph of the function $f(x) = \frac{x-6}{x-18}$ by determining intervals of increase and decrease, relative extrema, concavity, infection points and horizontal/vertical asymptotes (if any).

   Determine whether the graph of the given function has a vertical tangent or a cusp. If yes, find the same:

   $$f(x) = (4x-1)^{3/4} + 3$$

   It is estimated that when x units of a certain commodity are produced, the total cost (in rupees) is given by $C(x) = \frac{x^2}{2} + 16x + 800$ and they can all be sold at a price of (150 – x) rupees per unit. Find the level of production that would minimize the average cost.

2. Identify and sketch the curve: $3x^2 + y^2 + 2\sqrt{3}\,x - 8x + 8\sqrt{3}\,y = 0$.

   Sketch the polar curve: $r = 3cos3\theta$.

   Evaluate $\lim_{x\to\infty}[\frac{e^{5x}}{cosh5x}]$.

3. Find the length of the arc of the curve $y = \log\left[\frac{e^x-1}{e^x+1}\right]$ $from\ x = 1\ to\ x = 2$.

   Find the area of surface generated by revolving the curve $x = 3 + 2t\,, y = 9 - 3t\,; 1 \le t \le 4$ about $y - axis$.

   Let R be the region between the curves $y^2 = x^3\ and\ x^2 = y^3$. Find the volume of the solid generated when R is revolved about $x - axis$.

4. A ball is thrown from the ground level so as to just clear a wall 4 meters high at a distance of 4 meters from the point of projection and falling at a distance of 14 meters from the wall. Find the magnitude and direction of the velocity of the projection of the ball with the horizontal.

A particle moves in space with acceleration $\vec{\boldsymbol{a}}(t) = e^t\,\hat{\boldsymbol{\imath}} + 2\,t\,\hat{\boldsymbol{\jmath}} - 2\,sin\,2t\,\hat{\boldsymbol{k}}$. Find the particle's position if the position at time t=0 is at the point (2, 1, -1) and $\vec{\boldsymbol{v}}(0) = \hat{\boldsymbol{\imath}} + \hat{\boldsymbol{k}}$.

Find the arc length parametrization of the line $x = -5 + 3t,\ y = 2t,\ z = 5 + t$ that has the same direction as the given line and has reference point (-5, 0, 5).

5. Find the curvature and radius of curvature for the graph of vector equation

$\vec{\boldsymbol{r}}(t) = e^t\,cos\,t\,\hat{\boldsymbol{\imath}} + e^t\,sin\,t\,\hat{\boldsymbol{\jmath}} + e^t\,\hat{\boldsymbol{k}}$ at t=0.

Determine for what values of t the vector valued function

$\vec{f}(t) = \langle\, ln(t+1),\ |t+2|,\ [t]\,\rangle$ is continuous?

If the foci of an ellipse are at points (3,1) and (3, -5) with length of major axis as 12, find its equation.

6. If $\vec{R}(t)$ is the position vector of a particle in a plane at time t, find the time in the given interval when the velocity and acceleration are orthogonal, where

$\vec{R}(t) = (2t - 5\text{sint})\hat{\imath} + (2 - 5cost)\hat{\jmath}$; $0 \le t \le 2\pi$.

Evaluate the integral $\int_{-\pi/4}^{\pi/4} 6tan^4x\ dx$ using the Reduction formula.

If $q = tan^{-1}p$, prove that $(1+p^2)q_{n+2} + 2(n+1)pq_{n+1} + n(n+1)q_n = 0$.